MATH 208

SECTION 1.2

SECTION SUMMARY

In this section, we solve differential equations of the form

$$\frac{dy}{dx} = f(x) \qquad (1)$$

and $$\frac{d^2y}{dx^2} = g(x) \qquad (2)$$

where f and g are functions of the independent variable x. If we integrate both sides of equation (1) we get

$$y(x) = \int f(x) \; dx \; + C \qquad (3)$$

Equation (3) is called one-parameter family of solutions of the DE (1). We said this because this equation can generate **particular solutions** of the DE (1) with each choice of the parameter (constant) C.

The general solution of the differential equation $\frac{d^2y}{dx^2} = g(x)$ is $y = \int G(x) \; dx \; + C_1 x \; + C_2$

where $G(x)$ is any antiderivative of $g(x)$, that is $G(x) = \int g(x) \; dx$, C_1 and C_2 are arbitrary constants.

- Velocity and Acceleration

The motion of a particle along a straight line (the x −axis) is described by its **position function** $x = f(t)$

giving its x −coordinate at time t. The velocity of the particle is defined to be $v(t) = f'(t)$; that is, $v = \frac{dx}{dt}$.

Its acceleration $a(t)$ is $a(t) = v'(t) = x''(t)$; in Leibniz notation, $a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$.

A basic Formula

For $a > 0$,

$$\int \frac{1}{x^2 + a^2} \, dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{1}{\sqrt{1 - x^2}} \, dx = \sin^{-1} x + C$$

3. If the acceleration of a particle is $a(t) = 18\cos(3t)$, the initial velocity is $v_0 = 4$, and the initial position is $x_0 = -7$, then $x\left(\frac{\pi}{2}\right) =$

(a) $2\pi - 5$

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12. If the acceleration of a particle is $a(t) = \dfrac{1}{(t+1)^3}$, the initial velocity is $v_0 = 0$, and the initial position is $x_0 = 0$, then $x(1) =$

(a) $\dfrac{1}{4}$

8. A particle is moving in a straight line with acceleration $a(t) = 4(t+3)^2$, and an initial position $x(0) = 1$, and an initial velocity $v(0) = -1$, then the position function $x(t)$ of the particle is given by

(a) $x(t) = \frac{1}{3}(t+3)^4 - 37t - 26$

7. A particle is moving in a straight line with acceleration $a(t) = \dfrac{1}{\sqrt{t+9}}$, and initial position $x(0) = 4$, and an initial velocity $v(0) = 2$, then $x(16)$ (the position of the particle at $t = 16$) is

(a) $\dfrac{212}{3}$

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3. If $y = y(x)$ is the solution of the initial-value problem $\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$, $y(0) = 0$, then $y\left(\frac{1}{2}\right) =$

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}x + C$$

(a) $\frac{\pi}{6}$

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4. If a particle is moving in a straight line with acceleration $a(t) = \frac{1}{(t+1)^3}$, initial-position $x(0) = 0$, and an initial velocity $v(0) = 0$, then the position of the particle at $t = \frac{1}{2}$ is

(a) $\frac{1}{12}$

3. A particle is moving in a straight line with acceleration $a(t) = 18\cos(3t)$, and initial position $x(0) = -7$, and an initial velocity $v(0) = 4$.

Find $x(\pi)$ (the position of the particle at $t = \pi$).

(a) $4\pi - 3$

2. A particle is moving in a straight line with acceleration

$$a(t) = 12(t+1)^2,$$

an initial position $x(0) = 4$, and an initial velocity $v(0) = 5$. Find $x(1)$ (the position of the particle at $t = 1$).

(a) 20

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2. If $y = y(x)$ is the solution of the initial-value problem

$$\frac{dy}{dx} = x\sqrt{x^2+9}, \quad y(-4) = 0$$

, then $y(4) =$

(a) 0

2. A particle is moving along a straight line with position function $x(t)$. If the acceleration of the particle is $a(t) = 50\sin(5t)$, the initial position is $x(0) = 8$, and the initial velocity is $v(0) = -10$, then $x\left(\frac{\pi}{2}\right) =$

(a) 6 ____ (correct)

(b) 10

(c) 8

(d) 12

(e) 4

THANKS