MATH 208

SECTION 1.2

SECTION SUMMARY

$a(t)$

$v(t) = \int a(t) \cdot dt + C$

$x(t) = \int v(t) + C$

In this section, we solve differential equations of the form

$$\frac{dy}{dx} = f(x) \qquad (1)$$

$y' = f(x)$

and

$$\frac{d^2y}{dx^2} = g(x) \qquad (2)$$

$y'' = g(x)$

$a(t) = x''(t)$

$v(t) = x'(t)$

where f and g are functions of the independent variable x. If we integrate both sides of equation (1) we get

$$y(x) = \int f(x)\ dx + C \qquad (3)$$

Equation (3) is called one-parameter family of solutions of the DE (1). We said this because this equation can generate **particular solutions** of the DE (1) with each choice of the parameter (constant) C.

The general solution of the differential equation $\frac{d^2y}{dx^2} = g(x)$ is $y = \int G(x)\ dx + C_1 x + C_2$

where $G(x)$ is any antiderivative of $g(x)$, that is $G(x) = \int g(x)\ dx$, C_1 and C_2 are arbitrary constants.

- **Velocity and Acceleration**

The motion of a particle along a straight line (the x –axis) is described by its **position function** $x = f(t)$ giving its x –coordinate at time t. The velocity of the particle is defined to be $v(t) = f'(t)$; that is, $v = \frac{dx}{dt}$.

Its acceleration $a(t)$ is $a(t) = v'(t) = x''(t)$; in Leibniz notation, $a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$.

A basic Formula

For $a > 0$,

$$\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}x + C$$

3. If the acceleration of a particle is $a(t) = 18\cos(3t)$, the initial velocity is $v_0 = 4$, and the initial position is $x_0 = -7$, then $x\left(\frac{\pi}{2}\right) =$

$x(0) = -7$ $\quad$ $v(0) = 4$

$$\overset{x'(t)}{x''(t) = 18\cos(3t)},\ v(0) = 4,\ \underline{x(0) = -7},\ \underline{x\left(\tfrac{\pi}{2}\right) = ?}$$

$$\int x''(t)\cdot dt = x'(t) = 6\sin(3t) + 4$$

$$\int x'(t)\cdot dt = x(t) = -2\cos(3t) + 4t - 5$$

$$x\left(\frac{\pi}{2}\right) = 2\pi - 5$$

(a) $2\pi - 5$

Question 17 / Section 1.2 (Page 29)

12. If the acceleration of a particle is $a(t) = \dfrac{1}{(t+1)^3}$, the initial velocity is $v_0 = 0$, and the initial position is $x_0 = 0$, then $x(1) =$

$v(0) = 0$

$x(0) = 0$

$x''(t) = (t+1)^{-3}$

$x'(t) = \dfrac{(t+1)^{-2}}{-2} + \dfrac{1}{2}$

$x(1) = \dfrac{1}{4} + \dfrac{1}{2} - \dfrac{1}{2}$

$x(1) = \dfrac{3}{4} - \dfrac{2}{4} = \dfrac{1}{4}$

(a) $\dfrac{1}{4}$

8. A particle is moving in a straight line with acceleration $a(t) = 4(t+3)^2$, and an initial position $x(0) = 1$, and an initial velocity $\boxed{v(0) = -1}$, then the position function $x(t)$ of the particle is given by

$$x''(t) = 4(t+3)^2$$

$$x(0) = 1$$

$$v(t) = x'(t) = \frac{4}{3}(t+3)^3 - 37$$

$$\boxed{x(t) = \frac{1}{3}(t+3)^4 - 37t - 26}$$

(a) $x(t) = \frac{1}{3}(t+3)^4 - 37t - 26$ ✓

7. A particle is moving in a straight line with acceleration $a(t) = \dfrac{1}{\sqrt{t+9}}$, and initial position $x(0) = 4$, and an initial velocity $v(0) = 2$, then $x(16)$ (the position of the particle at $t = 16$) is

$x''(t) = (t+9)^{-\frac{1}{2}}$

$x'(t) = 2(t+9)^{\frac{1}{2}} - 4$

$x(t) = \frac{4}{3}(t+9)^{\frac{3}{2}} - 4t - 32$

$x(t) = \frac{500}{3} - \frac{288}{3}$

$x(t) = \frac{212}{3}$

500
288

(a) $\dfrac{212}{3}$

Question 9 / Section 1.2

3. If $y = y(x)$ is the solution of the initial-value problem $\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$, $y(0) = 0$, then $y\left(\frac{1}{2}\right) =$

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}x + C$$

$$y' = \frac{1}{\sqrt{1-x^2}}$$

$$\int y' \cdot dx = \sin^{-1}(x)$$

$$y(x) = \sin^{-1}(x)$$

$$\sin^{-1}\left(\frac{1}{2}\right)$$

$$\sin(?) = \frac{1}{2}$$

$$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$$

(a) $\frac{\pi}{6}$

Question 17 / Section 1.2

4. If a particle is moving in a straight line with acceleration $a(t) = \frac{1}{(t+1)^3}$, initial-position $x(0) = 0$, and an initial velocity $v(0) = 0$, then the position of the particle at $t = \frac{1}{2}$ is

$$x''(t) = (t+1)^{-3}$$

$$x'(t) = \frac{(t+1)^{-2}}{-2} + \frac{1}{2}$$

$$x(t) = \frac{1}{2(t+1)} + \frac{1}{2}t - \frac{1}{2}$$

$$x\left(\frac{1}{2}\right) = \frac{4\times 1}{4\times 3} + \frac{1\times 3}{4\times 3} - \frac{1\times 6}{2\times 6} = \frac{4+3-6}{12}$$

(a) $\frac{1}{12}$

3. A particle is moving in a straight line with acceleration $a(t) = 18\cos(3t)$, and initial position $x(0) = -7$, and an initial velocity $v(0) = 4$.

Find $x(\pi)$ (the position of the particle at $t = \pi$).

$$x''(t) = 18\cos(3t)$$

$$x'(t) = 6\sin(3t) + 4$$

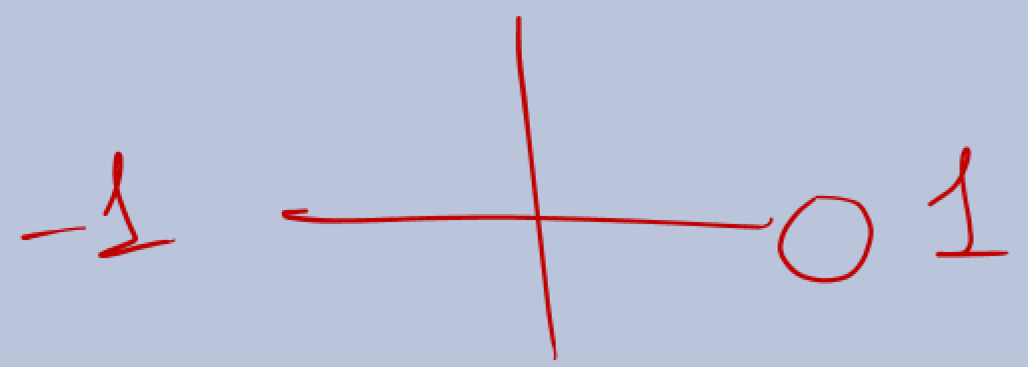
$$x(t) = -2\cos(3t) + 4t - 5$$

$$x(\pi) = -2\cos(3\pi) + 4\pi - 5$$

$$x(\pi) = (-2)\times(-1) + 4\pi - 5$$

$$x(\pi) = 2 + 4\pi - 5$$

$$x(\pi) = 4\pi - 3$$



(a) $4\pi - 3$

2. A particle is moving in a straight line with acceleration

$$a(t) = 12(t+1)^2,$$

an initial position $x(0) = 4$, and an initial velocity $v(0) = 5$. Find $x(1)$ (the position of the particle at $t = 1$).

$$x''(t) = 12(t+1)^2$$

$$x'(t) = 4(t+1)^3 + 1$$

$$x(t) = (t+1)^4 + t + 3$$

$$x(t) = 16 + 1 + 3 = 16 + 4 = 20$$

(a) 20

Q6/ P. 17 (Section 1.2)

2. If $y = y(x)$ is the solution of the initial-value problem

$$\frac{dy}{dx} = x\sqrt{x^2+9}, \quad y(-4) = 0$$

, then $y(4) =$

$y' = x\sqrt{x^2+9}$

$y' = \frac{1}{2}\int u^{\frac{1}{2}}\,du$

$y = \frac{1}{3}u^{\frac{3}{2}} + C$

$y = \frac{1}{3}(x^2+9)^{3/2} - \frac{125}{3}$

$y = \frac{125}{3} - \frac{125}{3}$

$y(4) = 0$

(a) 0

2. A particle is moving along a straight line with position function $x(t)$. If the acceleration of the particle is $a(t) = 50\sin(5t)$, the initial position is $x(0) = 8$, and the initial velocity is $v(0) = -10$, then $x\left(\frac{\pi}{2}\right) =$

(a) 6 (correct)

(b) 10

(c) 8

(d) 12

(e) 4

THANKS