Chapter1: Equations and Inequalities.

1.4: Complex Numbers

The Imaginary Unit i

The imaginary unit i is defined as

$$i = \sqrt{-1}, \quad \text{where } i^2 = -1$$

Example1

$\sqrt{-25} =$ $\sqrt{-16} =$ $\sqrt{-9} =$ $\sqrt{-5} =$ $\sqrt{-3} =$

$$\sqrt{-a} = \sqrt{a}\, i$$

$(-i)^2 =$ $(2i)^2 =$ $(-2i)^2 =$ $(3i)^2 =$ $(-3i)^2 =$

$$(ai)^2 = (-ai)^2 = -a^2$$

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Complex Numbers and Imaginary Numbers

The set of all numbers in the form

$$a + bi$$

with real numbers a and b, and i, the imaginary unit, is called the set of ***complex numbers***. "The real number a is called the ***real part*** and the real number b is called the ***imaginary part*** of the complex number $a + bi$.

If $b \neq 0$, then the complex number is called an ***imaginary number***

An imaginary number in the form bi is called a ***pure imaginary number.***

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Example2 The complex number

$$3 - 4i \quad \text{has}$$

3 as the *real part* and -4 as the *imaginary part.*

$3 - 4i$ is an *imaginary number,*

$7i$ is a *pure imaginary number.*

They are all called complex numbers.

The complex number $5i$ is the same as

$$0 + 5i$$

0 is the *real part* and 5 is the *imaginary part.*

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Conjugate of a Complex Number

The ***complex conjugate*** of the number $a + bi$ is $a - bi$, and

the ***complex conjugate*** of the number $a - bi$ is $a + bi$

Example3

The complex conjugate of $3 - 5i$ is $3 + 5i$.

The complex conjugate of $7 + 2i$ is $7 - 2i$.

The complex conjugate of $8i$ is $-8i$.

The complex conjugate of 4 is 4

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Example4 Find the following:

Complex number	Real part	Imaginary	Conjugate
$2 - 7i$			
$-4i$			
$\sqrt{2}$			
$\frac{1}{4} - \frac{3}{4}i$			
$\sqrt{-9}$			

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Adding and Subtracting Complex Numbers.

$$(a+bi)+(c+di)=(a+c)+(b+d)i$$

$$(a+bi)-(c+di)=(a-c)+(b-d)i$$

Example5 Find the following:

1) $(7+2i)+(1-4i)$	3) $(3+2i)-(5-7i)$

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5) $6-(-5+4i)-(-13-i)$	4) $(5-2i)-(3-4i)$

Multiplying Complex Numbers:

$$(a+bi)(c+di) = ac+adi+bci+bdi^2$$
$$= ac+adi+bci-bd$$
$$= ac-bd+(ad+bc)i$$

Multiplication of conjugates: $(a+bi)(a-bi) = a^2+b^2$

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Example6 Find the following:

11) $(-5+4i)(3+i)$	**15)** $(3+5i)(3-5i)$
19) $(2+3i)^2 =$	**20)** $(2i-3)^2 =$

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Dividing Complex Numbers:

$$\frac{a+bi}{c+di}=\frac{a+bi}{c+di}\cdot\frac{c-di}{c-di}=\frac{(a+bi)(c-di)}{c^2+d^2}$$

Example7 Perform the indicated operations and write the result in standard form:

21) $\frac{2}{3-i}=$	**27)** $\frac{2+3i}{2+i}=$

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31) $5\sqrt{-16} + 3\sqrt{-81}$	**32)** $(2 - \sqrt{-7})^2 =$	**39)** $\dfrac{-6 - \sqrt{-12}}{48}$

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41) $\sqrt{-8}(\sqrt{-3}-\sqrt{5})$	42) $\sqrt{-9}\sqrt{-4}-\sqrt[3]{-27}$	43) $\frac{(3-4i)(3+4i)}{10-5i}$

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44) $\frac{2-3i}{1-2i}+\frac{\sqrt{-36}}{\sqrt{-4}\sqrt{-9}}$	45) $\frac{1-3i}{1+i}$	46) $2i(2-3i)^2=$

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