Chapter4: Exponential and Logarithmic Functions
4.1 : Exponential Functions

Definition:

The ***exponential function*** f with base $\boldsymbol{b}$ is defined by $\boldsymbol{f(x) = b^x}$ ***or*** $\boldsymbol{y = b^x}$ where **b** is a positive constant other than 1. ($\boldsymbol{b} > 0$, $b \neq 1$).

x is any real number.

Domain of exponential function $\boldsymbol{f(x) = b^x}$: all real numbers (R).

Range of exponential function $\boldsymbol{f(x) = b^x}$: $(0, \infty)$

Examples :

$f(x) = 2^x$, $g(x) = 10^x$, $h(x) = \pi^x$, $j(x) = \left(\frac{1}{2}\right)^{x-1}$, $k(x) = 3^{-x+1}$

standard form transformed form

The function $\boldsymbol{f(x) = e^x}$ is called a ***natural exponential function***. The irrational number $e \approx 2.72$ is called a ***natural base***.

Examples of non exponential functions: x

$g(x) = (-1)^x$, $f(x) = x^x$, $k(x) = 1^x$, $g(x) = (-4)^x$, $H(x) = x^2$

negative number variable base = 1 negative number variable

➤ **Evaluating an exponential function:**

Let $g(x) = (1.56)^x$ evaluate $g(4) = (1.56)^4$ = 5.922

Example 1: Approximate each number using a calculator . *Round your answer to three decimal places*

5) $4^{-1.5} =$ 0.125

9) $e^{-0.95} =$ 0.3867 ≈ 0.387

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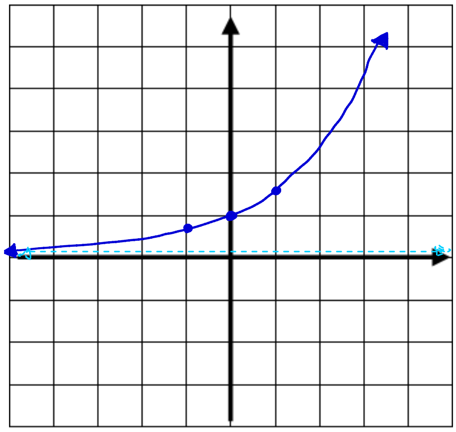
- **Graphing Exponential Functions**:

Example 2:

13) Graph $f(x) = \left(\frac{3}{2}\right)^x$. Then find domain, range and the equation of asymptote.

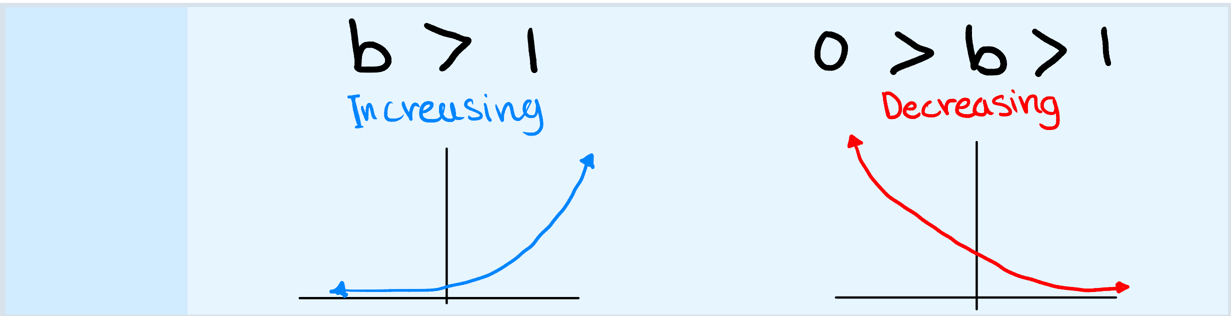
standard form

x	$f(x)$	(x, y)
-1	$\left(\frac{3}{2}\right)^x = \left(\frac{3}{2}\right)^{-1} = 0.7$	$(-1, 0.7)$
0	$\left(\frac{3}{2}\right)^x = \left(\frac{3}{2}\right)^0 = 1$	$(0, 1)$
1	$\left(\frac{3}{2}\right)^x = \left(\frac{3}{2}\right)^1 = 1.5$	$(1, 1.5)$



- The graph is increasing because $b > 1$
- Asymptote

$f(x)$	
Domain	$(-\infty, \infty)$ or $\mathbb{R}$
Range	$(0, \infty)$
Horizontal Asymptote H.A	on the x-axis $y = 0$



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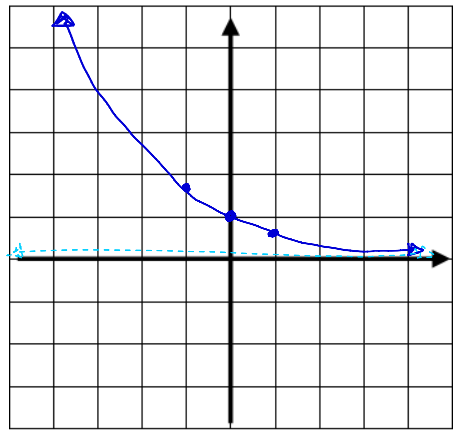
17) Graph $f(x) = (0.6)^x$. Then find domain, range and the equation of asymptote

x	$f(x)$	(x, y)
-1	$(0.6)^x = (0.6)^{-1} = 1.66$	$(-1, 1.66)$
0	$(0.6)^x = (0.6)^0 = 1$	$(0, 1)$
1	$(0.6)^x = (0.6)^1 = 0.6$	$(1, 0.6)$



• The graph is decreasing because $0 > b > 1$

$f(x)$	
Domain	$(-\infty, \infty)$ or $\mathbb{R}$
Range	$(0, \infty)$
Horizontal Asymptote H.A	on the x-axis $y=0$

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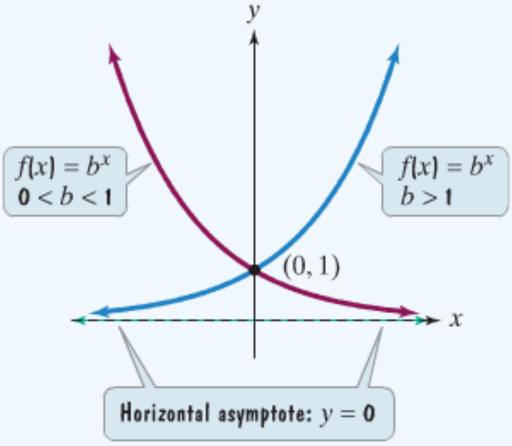
Characteristics of Exponential Functions of the Form $f(x) = b^x$

1. The domain of $f(x) = b^x$ consists of all real numbers: $(-\infty, \infty)$. The range of $f(x) = b^x$ consists of all positive real numbers: $(0, \infty)$.
2. The graphs of all exponential functions of the form $f(x) = b^x$ pass through the point $(0, 1)$ because $f(0) = b^0 = 1$ $(b \neq 0)$. The y-intercept is 1. There is no x-intercept.
3. If $b > 1$, $f(x) = b^x$ has a graph that goes up to the right and is an increasing function. The greater the value of b, the steeper the increase.
4. If $0 < b < 1$, $f(x) = b^x$ has a graph that goes down to the right and is a decreasing function. The smaller the value of b, the steeper the decrease.
5. $f(x) = b^x$ is one-to-one and has an inverse that is a function.
6. The graph of $f(x) = b^x$ approaches, but does not touch, the x-axis. The x-axis, or $y = 0$, is a horizontal asymptote.



Transformation of Exponential Function $(f(x) = b^x)$

Transformation	Equation	Description
Vertical Shift shift in y	$g(x) = b^x + c$ ↑up	$(x, y) \to (x, y + c)$
	$g(x) = b^x - c$ ↓down	$(x, y) \to (x, y - c)$
Horizontal Shift shift in x	$g(x) = b^{x+c}$ ← left	$(x, y) \to (x - c, y)$
	$g(x) = b^{x-c}$ → right	$(x, y) \to (x + c, y)$
Reflection about x-axis (-y)	$g(x) = -b^x$	$(x, y) \to (x, -y)$
Reflection about y-axis (- x)	$g(x) = b^{-x}$	$(x, y) \to (-x, y)$
Vertical stretching or shrinking	$g(x) = cb^x$	$(x, y) \to (x, cy)$
Horizontal stretching or shrinking	$g(x) = b^{cx}$	$(x, y) \to (\frac{x}{c}, y)$

Transformations

Shifts

Horizauntly

Shift in x

+ Left

− right

Vertically

shift in y

+ Up

− down

Reflection

Reflection about x-axis

$-y$

$g(x) = -f(x)$

$\underset{f(x)}{(x, y)} \longrightarrow \underset{g(x)}{(x, -y)}$

Reflection about y-axis

$g(x) = f(-x)$

$-x$

$\underset{f(x)}{(x, y)} \longrightarrow \underset{g(x)}{(-x, y)}$

Vertically Stretching Shrinking Graphs

$0 > C > 1$

Stretch

Multipling y by C ← C: any number

$\underset{f(x)}{(x, y)} \longrightarrow \underset{g(x)}{(x, C \cdot f)}$

$0 < C < 1$

Shrink

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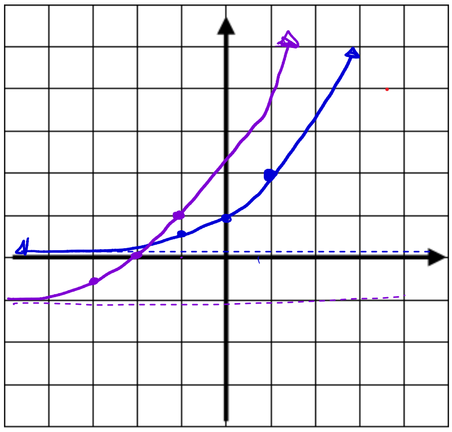
Example3:

29) Begin by Graphing $f(x) = 2^x$.then use transformation of this graph to graph given function. Give the equation of the asymptotes. Use the graph to determine each function's domain and range

a) $h(x) = 2^{x+2} - 1$



x	$f(x) = 2^x$	(x, y)	$h(x) = 2^{x+2} - 1$ $(x-2, y-1)$
-1	$2^{-1} = \frac{1}{2}$	$(-1, \frac{1}{2})$	$(-1-2, \frac{1}{2}-1)$ $(-3, -\frac{1}{2})$
0	$2^0 = 1$	$(0, 1)$	$(0-2, 1-1)$ $(-2, 0)$
1	$2^1 = 2$	$(1, 2)$	$(1-2, 2-1)$ $(-1, 1)$

$f(x) = 2^x$	
Domain	$(-\infty, \infty)$
Range	$(0, \infty)$
Horizontal Asymptote H.A	$y = 0$

$h(x) = 2^{x+2} - 1$	
Domain	$(-\infty, \infty)$
Range	$(-1, \infty)$
Horizontal Asymptote H.A	$y = -1$

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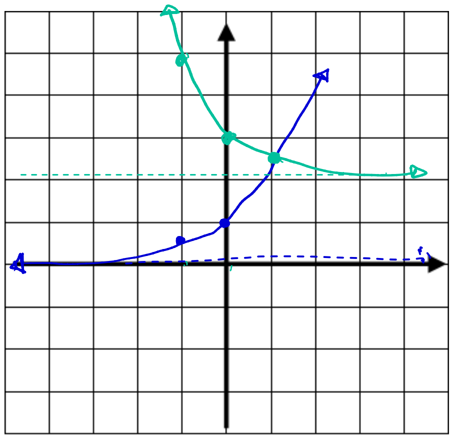
Example 4: Begin by Graphing $f(x) = e^x$.then use transformation of this graph to graph given function. Give the equation of the asymptotes. Use the graph to determine each function's domain and range

$g(x) = e^{-x} + 2$

x	$f(x) = e^x$	(x, y)	$(-x, y+2)$
-1	$e^{1} = 0.4$	$(-1, 0.4)$	$(1, 2.4)$
0	$e^0 = 1$	$(0, 1)$	$(0, 3)$
1	$e^1 = 2.4$	$(1, 2.4)$	$(-1, 4.7)$



$g(x) = e^{-x} + 2$	
Domain	$(-\infty, \infty)$
Range	$(2, \infty)$
Horizontal Asymptote H.A	$y = 2$

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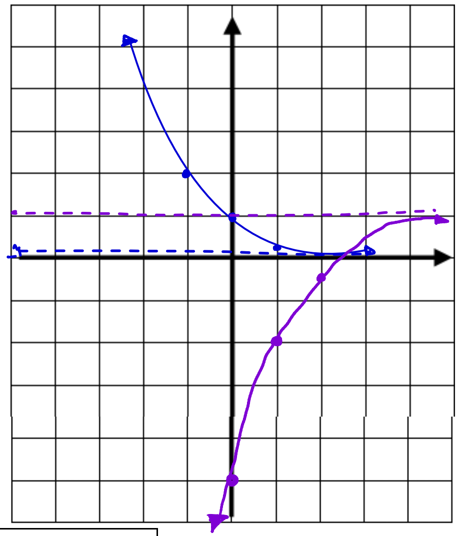
Example5:

Begin by Graphing $f(x) = \left(\frac{1}{2}\right)^{x}$.then use transformation of this graph to graph given function. Give the equation of the asymptotes. Use the graph to determine each function's domain and range

$$g(x) = -3\left(\frac{1}{2}\right)^{x-1} + 1$$

x	$f(x)$	(x,y)	$(x+1\ ,-3y+1)$
-1	$\left(\frac{1}{2}\right)^{-1} = 2$	$(-1, 2)$	$(-1+1\ ,-3(2)+1)$ $(0\ ,-5)$
0	$\left(\frac{1}{2}\right)^{0} = 1$	$(0, 1)$	$(0+1\ ,-3(1)+1)$ $(1\ ,-2)$
1	$\left(\frac{1}{2}\right)^{1} = \frac{1}{2}$	$(1, \frac{1}{2})$	$(1+1\ ,-3(\frac{1}{2})+1)$ $(2\ ,-0.5)$



$f(x)$	
Domain	$(-\infty, \infty)$
Range	$(0, \infty)$
Horizontal Asymptote H.A	$y=0$

$g(x)$	
Domain	$(-\infty, \infty)$
Range	$(-\infty, 1)$
Horizontal Asymptote H.A	$-3(0)+1=1$ $y=1$

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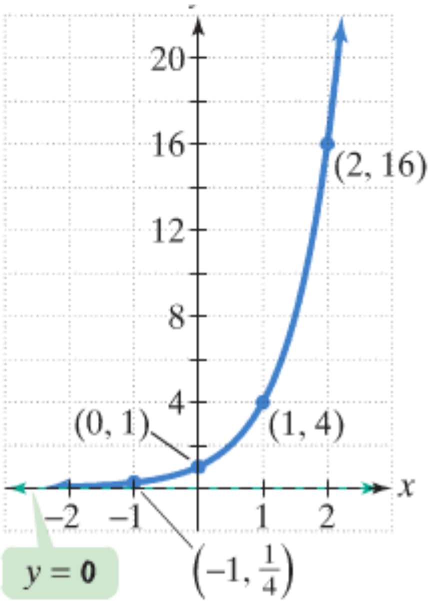
Example 6:

61) Give the equation of the exponential function whose graph is shown.



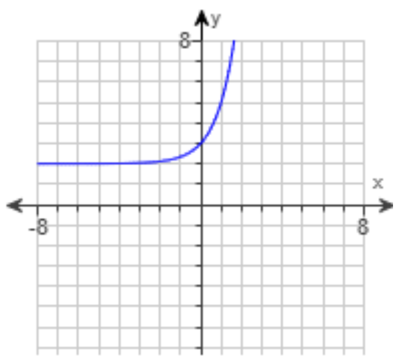
- The standard form $f(x) = b^x$ or $y = b^x$
- $f(x) = b^x = 4 = b^{(1)} \rightarrow 4 = b$

  using The point $(1, 4)$
- The final answer is $\boxed{f(x) = 4^x}$

Example:

The graph of an exponential function is given. Select the functio
from the functions listed.

- A. $f(x) = 3^x$
- B. $f(x) = 3^{x+2}$
- C. $f(x) = 3^x - 2$
- D. $f(x) = 3^x + 2$



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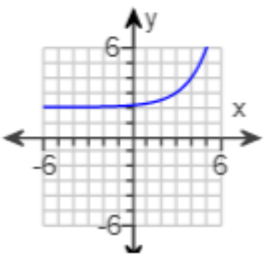
Example

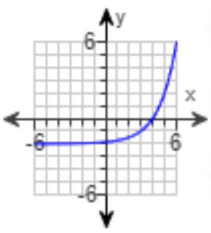
Graph the function.

Use the graph of $f(x) = 2^x$ to obtain the graph of $g(x) = 2^{x+3} + 2$.

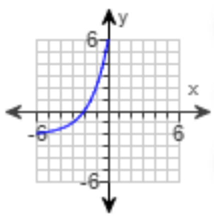
○ A.



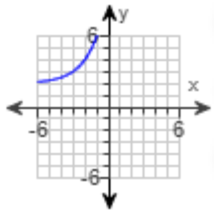
○ B.



○ C.



● D.



Extra: Find domain, Range and asymptote

- $g(x) = -e^{x+3} + 1$

  Domain: $(-\infty, \infty)$ or $\mathbb{R}$

  Range: $(1, \infty)$

  Asymptote: $y = 1$

- $g(x) = \frac{1}{2}(2)^{x-1} + 3$

  Domain: $(-\infty, \infty)$ or $\mathbb{R}$

  Range: $(3, \infty)$

  Asymptote: $y = 3$

Use a calculator with a $\boxed{y^x}$ *key or a* $\boxed{\wedge}$ *key to solve Exercises 65–70.*

65. India is currently one of the world's fastest-growing countries. By 2040, the population of India will be larger than the population of China; by 2050, nearly one-third of the world's population will live in these two countries alone. The exponential function $f(x) = 574(1.026)^x$ models the population of India, $f(x)$, in millions, x years after 1974.

a. Substitute 0 for x and, without using a calculator, find India's population in 1974. $f(x) = 574(1.026)^0 = 574$

b. Substitute 27 for x and use your calculator to find India's population, to the nearest million, in the year 2001 as modeled by this function. $f(x) = 574(1.026)^{27} \rightarrow 1,148$ without decimals $2001 - 1974 \rightarrow x = 27$

c. Find India's population, to the nearest million, in the year 2028 as predicted by this function.
$f(x) = 574(1.026)^{54} \rightarrow 2295$ $2028 - 1974 \rightarrow x = 54$

d. Find India's population, to the nearest million, in the year 2055 as predicted by this function.
$f(x) = 574(1.026)^{81} \rightarrow 4590$ $2055 - 1974 \rightarrow x = 81$

e. What appears to be happening to India's population every 27 years?

- Doubles up (get multiplied by 2)
- Increases by the double

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73. In college, we study large volumes of information—information that, unfortunately, we do not often retain for very long. The function

$$f(x) = 80e^{-0.5x} + 20$$

describes the percentage of information, $f(x)$, that a particular person remembers x weeks after learning the information.

a. Substitute 0 for x and, without using a calculator, find the percentage of information remembered at the moment it is first learned. $f(x) = 80e^{-0.5(0)} + 20 \rightarrow 80(1) + 20 \rightarrow 100\,\%$

$x = 0$

b. Substitute 1 for x and find the percentage of information that is remembered after 1 week.

$x = 1$ $\quad f(x) = 80e^{-0.5(1)} + 20 \rightarrow 69\,\%$

c. Find the percentage of information that is remembered after 4 weeks.

$x = 4$ $\quad f(x) = 80e^{-0.5(4)} + 20 \rightarrow 31\,\%$

d. Find the percentage of information that is remembered after one year (52 weeks).

$x = 52$ $\quad f(x) = 80e^{-0.5(52)} + 20 \rightarrow 20\,\%$